



# Pavement service life prediction and inverse analysis with PLAXIS 3D

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Predicting the end of service life of an engineering structure, or obtaining parameters from inverse analysis of measured forces or displacements, is a complex task which requires deep knowledge of material behaviour and software development. In this paper it is shown how these two procedures can be carried out by writing a subroutine with the software MATLAB to run with predefined input data, pre- and post-processing a finite element model in PLAXIS 3D. The examples show how it is possible to predict the rest of service life of an airport pavement and to obtain layer stiffness parameters from inverse analysis of a three-dimensional deflection bowl. The developed MATLAB routines allow the field of possible PLAXIS 3D applications to be extended considerably.

➤ In civil engineering structures, failure occurs when internal stresses exceed their ultimate limit strength. In particular cases, when structures are subjected to repeated loading, failure occurs due to fatigue even if the stresses are much lower than the material strength. Fatigue phenomena can be observed in pavements or structures subjected to dynamic loading, e.g. fatigue of bridge elements. Fatigue is a very complex phenomenon in which material accumulates incremental structural damage due to repeated loading until it reaches failure. The physical damage is induced by micro-cracks that develop in the material. At a macroscopic scale, this means a significant reduction in stiffness. In order to design structures against fatigue, we therefore need to be able to predict the damage development and the consequent stiffness reduction during the service life.

Another important issue in the design of pavements or other geotechnical structures is the reliability of soil material parameters, in particular the stiffness. This paper shows how these parameters can be obtained from inverse analysis of pavement deflections and potentially from geotechnical measurements

## Service life prediction for pavements

The stress levels in a pavement structure are generally much lower than the failure values. Instead, pavement "failure" is due to accumulation of damage and is generally related to structure

serviceability, in particular the amount of cracks and rutting. In the following sections, the methods followed for assessing the development of damage and the consequent stiffness reduction are described.

## Assessment of damage

In the adopted method, damage is defined based on the number of cycles to failure  $N_f$  that are obtained as a result of fatigue tests.

The (incremental) damage  $\Delta D$  is obtained by dividing the number of cycles at each calculation step  $n$  (e.g. number of load applications in one month) by the number of cycles to failure

$$\Delta D = \frac{n}{N_f}$$

Since the number of load cycles to failure  $N_f$  is not constant, and it varies with temperature, material stiffness and strain amplitude, the fatigue characteristics have in principle to be determined

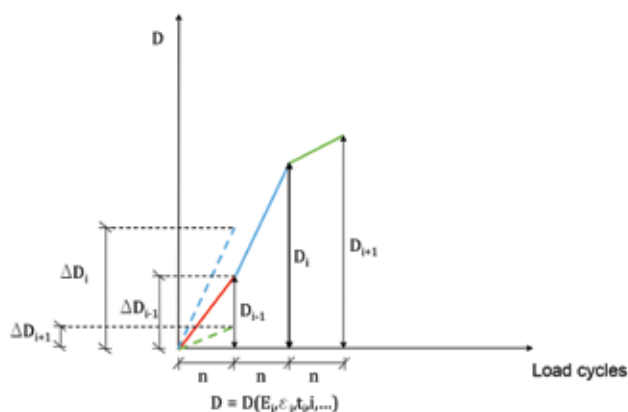


Figure 1: Incremental calculation and accumulation of damage according to Miner's rule. The incremental damage at the calculation step  $i - 1$  is equal to  $\Delta D_{i-1} = \frac{n}{N_{Fi-1}}$ . In the calculation step  $i$  the incremental damage becomes  $\Delta D_i = \frac{n}{N_{Fi}}$  where,  $N_{Fi-1} \neq N_{Fi}$ , and the total damage is equal to  $D = \sum_{i=1}^j \Delta D_i$  ( $t$  = temperature,  $i$  = increment,  $e$  = critical strain)



based on a large number of tests under different conditions. In this study, the number of cycles to failure is instead predicted according to the semi-empirical fatigue law that is described in the AASHTO 2008 pavement design guide for alligator cracking. This predictive equation depends on the tensile strain level  $\epsilon_{crit}$ , the elastic modulus  $E$  and empirical coefficients.

The total damage  $D$  is obtained by adding the increments  $D = \sum_{i=1}^n \Delta D_i$  after each iteration according to Miner's rule. Of course, this is a very simple model for the calculation of cumulative fatigue damage (see also Fatemi and Yang, 1998). This approach has been chosen because of its simplicity, and it is still widely accepted for practical applications. Despite its simple formulation, the calculated damage accumulation is non-linear: the incremental damage  $\Delta D_i$  is not constant (Figure 1), since  $N_{Fi}$  depends on the current stiffness and critical strains and climatic (temperature) conditions.

**Assessment of stiffness reduction due to damage**  
As already discussed, material damage is responsible for the reduction of stiffness. In literature there are several approaches to relate damage to stiffness (see also Collop and Cebon, 1995).

According to a well-known criterion, fatigue failure (100% damage) in asphalt is defined as the number of cycles to failure  $N_F$  after which the material stiffness (elastic modulus) reaches half of its initial value. Therefore, in the procedure adopted in this paper, the stiffness decay is obtained by multiplying the asphalt elastic modulus by  $(1 - \frac{D_i}{2})$ . The asphalt stiffness is additionally modified at each step based on average monthly layer temperature, according to the well-known Van der Pool monographs.

During each iteration, the calculated reduced stiffness is adopted as an input for a new finite element (FE) calculation, and the new critical strains in the asphalt layers are obtained. Once the new critical strains and current

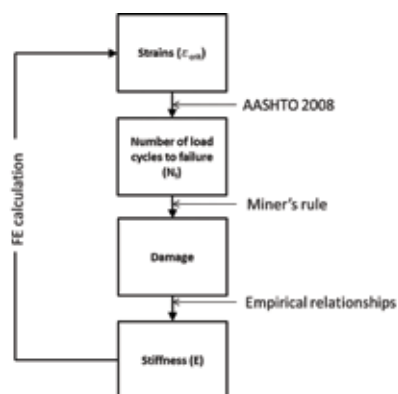


Figure 2: Simulation of the damage process due to fatigue in asphalt

stiffness have been obtained, a new value of load applications to failure  $N_{Fi}$  is predicted and the incremental damage  $\Delta D_i = \frac{n}{N_{Fi}}$  and the new stiffness can be assessed again.

The iteration procedure stops when the level of damage reaches 100%. The overall procedure is summarized in Figure 2.

#### Algorithm implementation

Predicting service life generally requires carrying out hundreds of calculations, one for each calculation step (e.g. 1 month). Therefore an automated procedure needs to be implemented. A major advantage of PLAXIS 3D is that the model can be pre- and post-processed and run under DOS. For pre-processing, a special .log file should be created by the MATLAB-based software with the instructions for the PLAXIS 3D command line. The coded output is then translated into a .txt file with the cbin.exe program. Special software has therefore been written in MATLAB that writes the PLAXIS 3D commands in the .log file and starts the calculation. The critical strains at the bottom of

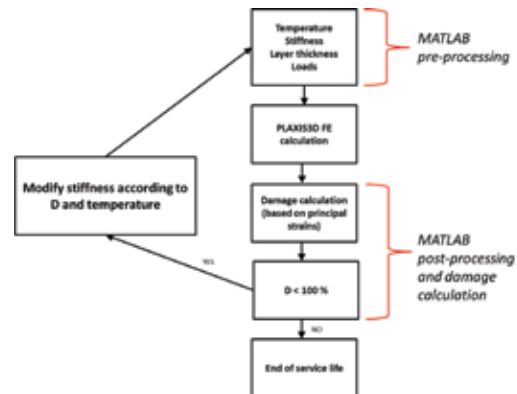


Figure 3: Implemented procedure for estimating service life (fatigue) with MATLAB (main routine) and PLAXIS 3D (subroutine)

the asphalt base layers are calculated in a three-dimensional finite element model implemented in PLAXIS 3D.

After the calculation finishes, MATLAB runs the translating output software cbin.exe and reads the stresses in the gauss points. MATLAB calculates the strains from the stresses according to the elastic constitutive model adopted for modelling the asphalt behaviour. The asphalt stiffness for the next calculation is obtained, as already mentioned, by considering the damage level and the average monthly temperature during service hours of the airport. The procedure is illustrated in Figure 3.

#### Assessment of service life for a rehabilitated pavement

The software has been successfully adopted for estimating the residual service life of the proposed rehabilitated pavement of runway 14/32 of the Zurich international airport. In this analysis, the ultimate goal was not to identify the period of time until 100% damage would be reached but to predict the level of damage after a period of 30 years.

The requirements for the service life duration were defined by the airport authorities. In the following paragraphs a summary of the most relevant information on the modelling is given. More details can be found in Rabaiotti et al. (2013).

The pavement and the loading caused by the landing gear were reproduced in PLAXIS 3D. Owing to the symmetry of the landing gear (Boeing B777-300ER), it is possible to model only one-quarter of the pavement, and the symmetry boundary conditions are applied accordingly. The model dimensions of 12 x 8 x 6 m (depth) were chosen in order to reduce the influence of the boundary conditions on the calculated results.

The same model is always loaded and unloaded with updated material properties. Thanks to this feature and the choice of the HSs model for the subgrade, it is therefore possible to follow accumulation of plastic strains (post-compaction) in the unbound (subgrade) layers.

In accordance with the airport's requirements, the performance of the rehabilitated pavement was studied on loading and unloading the finite element model (through the MATLAB routine) for an equivalent period of 30 years. After this time period, the damage was evaluated. Figures 5a and 5b show the development of the temperature-dependent asphalt stiffness and accumulation of

between calculated and measured displacements (Figure 6).

The procedure is implemented as follows: the MATLAB routine runs the cbin.exe program and translates the PLAXIS output to a readable .txt file. The calculated displacements are read and compared to those measured. The program calculates the objective function (SEE) and chooses a new set of parameters according to the chosen algorithm strategy. It then runs the next PLAXIS calculation with those parameters.

This procedure is carried out for several iterations until the value of the objective function is small enough. In the example presented in this paper, inverse analysis was carried out using ETH Delta

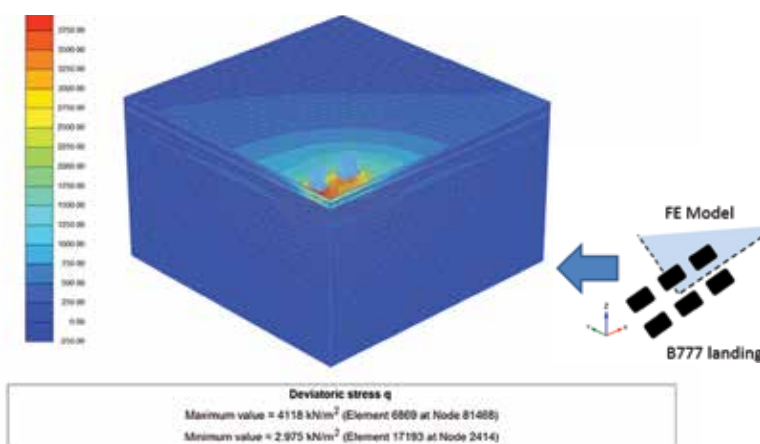


Figure 4a: Symmetrical 3D finite element model of 1/4 of the pavement and load (B777-300ER landing gear). The plot shows the calculated deviatoric stresses q

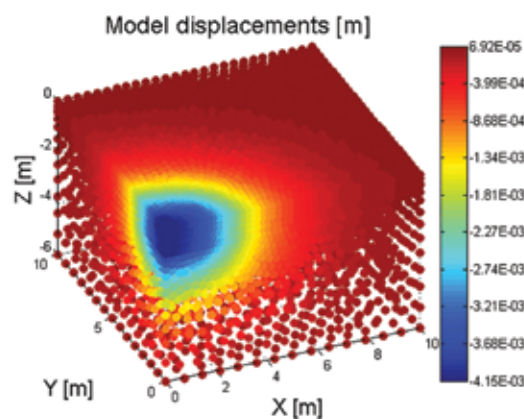


Figure 4b: MATLAB post-processing of pavement displacements calculated with PLAXIS 3D

A plot of the PLAXIS 3D model and an example of the MATLAB post-processing of the results are shown in Figures 4a and 4b.

The pavement consists of a layer of asphalt (wearing course and base), cement treated material (sub-base) and subgrade. An interface layer between base and sub-base layer was also modelled.

The asphalt and the cement treated base were modelled with a linear elastic constitutive model. The changes in the asphalt elastic modulus due to temperature and damage were calculated by the MATLAB routine within the previously described routine. The temperatures were obtained from measurements carried out in the layers of an instrumented track nearby, during the years 2003–2005 (Rabaiotti and Caprez, 2007).

To model the mechanical behaviour of the subgrade, the constitutive model chosen is the hardening-soil with small strain stiffness (HSs) model. Since the stiffness of the asphalt layers decreases because of damage during the service life, the compression stresses on the subgrade become higher. The increase in the stress level produces irreversible plastic strains in the subgrade; these can be simulated with the adopted HSs model.

In the implemented code, the results from each calculation step are stored for the next calculation; this means that during the simulation

damage during the service life. It was found that the proposed rehabilitation design fulfilled the requirements to last for 30 years.

**Inverse analysis: back-calculation of road material properties based on three-dimensional deflection bowl**

The procedure for back-calculating the stiffness of a pavement material layer based on a three-dimensional deflection bowl is extensively described in Rabaiotti (2008). The three-dimensional displacement of a pavement under a track load is measured with the ETH Delta test device. The depth and shape of the deflection bowl depend on the layer thickness and stiffness. If the thickness of the layers is known, it is possible to back-calculate the stiffness of the single layers within inverse analysis.

The inverse analysis is carried out by seeking the stiffness parameter values that allow a good match to be obtained between the measured and calculated displacements of the pavement. The parameters can be obtained with different strategies (gradient or non-gradient based optimization methods) that are already implemented in the MATLAB Optimization Toolbox. The inverse analysis procedure is carried out in a very similar manner to the simulation of the service life: the parameters (generally Young's moduli of the individual layers) are chosen by the algorithm in order to minimize the objective function  $f_{obj}$ , e.g. the sum of squared error (SSE)

measurements (see also Rabaiotti et al. 2013) on runway 16/34 of Zürich international airport. The runway was rehabilitated by replacing the old concrete slabs with an equivalent asphalt layer in 2008.

By observing the change of layer stiffness in different runway sections, in particular the heavily loaded initial part (threshold) and lightly loaded middle section, it was possible to quantify the development of the damage in the asphalt and cement-treated base layer. The results are extensively discussed in Rabaiotti et al. 2013. Figure 7 shows the match between back-calculated (lines) and measured (dots) transversal shape of the deflection bowl for different longitudinal wheel positions.

**Other possible applications**

The MATLAB software allows for a wide range of future possible applications to be developed. Inverse analysis can, of course, be extended to many geotechnical engineering problems, e.g. back-calculation of soil parameters from deformation of retaining walls of excavations. An interesting field could be the implementation of more advanced statistically based analysis of geotechnical or civil engineering structures, in which the input parameters or even the model geometry can be set according to statistically distributed values. The resulting distribution of the output, e.g. internal forces in the structure, could allow the safety of the structure to be statistically determined.

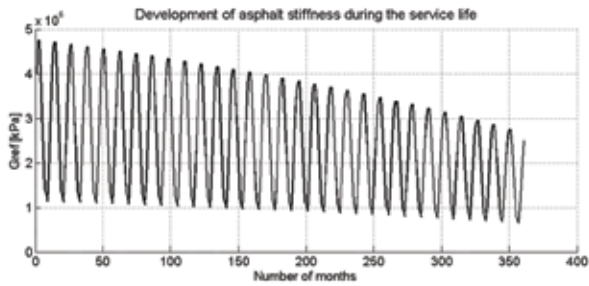


Figure 5a: Decrease of temperature-dependent asphalt stiffness (shear modulus G) during runway 14/32 service life



Figure 5b: Accumulation of damage during runway 14/32 service life

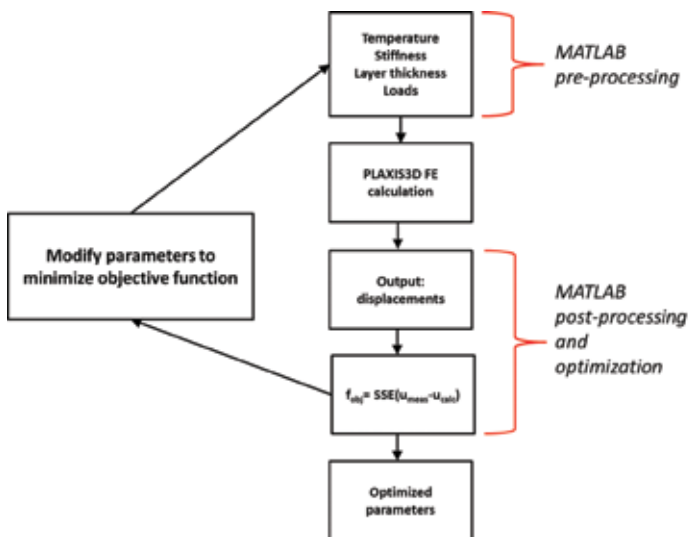


Figure 6: Inverse analysis procedure, coupling MATLAB (main routine) and PLAXIS 3D (subroutine)

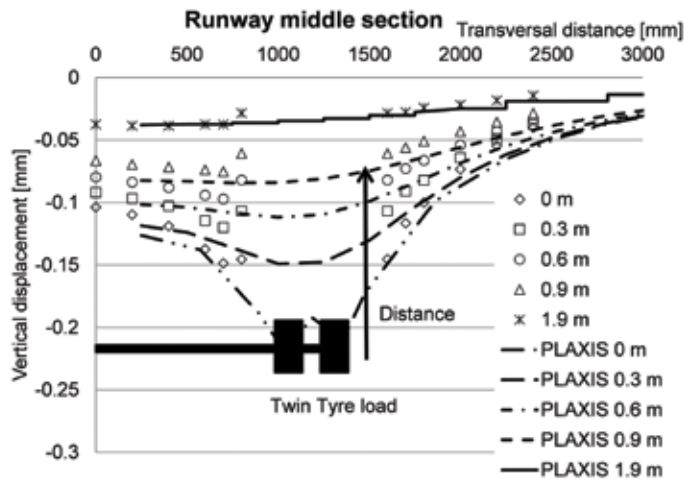


Figure 7: Measured (dots) and back-calculated (lines) deflection bowl induced by a track load (twin tyre). The test was carried out on runway 16/34 of Zürich international airport

**Conclusions**

Simulating the service life of structures subjected to repeated loading (fatigue) or using inverse analysis procedures to determine stiffness parameters, requires the implementation of predictive and optimization algorithms. In the present study, a service life prediction algorithm and an optimization procedure have been implemented in MATLAB.

PLAXIS3D was adopted as a subroutine to calculate the critical strains in the asphalt layer and to simulate the accumulation of damage. The same model, representing the rehabilitated pavement of runway 14/32 at Zürich international airport, was loaded and unloaded with changing asphalt stiffness for an equivalent period of 30 years.

It was shown that the proposed rehabilitation was able to fulfill the 30 years' service life requirements. Additionally, a modified version of the MATLAB routine was adopted for inverse analysis of ETH Delta measurements carried out on runway 16/34, which was rehabilitated with the same strategy in 2008. The results of the inverse

analysis allowed the development of the damage in the asphalt and cement-treated base layer to be quantified. Linking PLAXIS3D and MATLAB for pre- and post-processing considerably broadens the field of possible applications for finite element calculations. A desirable option for PLAXIS3D would be to obtain a written output of the strains, in order to avoid basing the calculation on nodal stresses or displacement.

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